MATHEMATICS AS A DISCIPLINE AND AS A SCHOOL COURSE: A SYSTEMIC COMPARATIVE STUDY CONDUCTED IN BRAZIL AND GREECE ABOUT THE IN-SERVICE SCHOOL PRINCIPALS' CONSTRUCTIONS

Andreas Moutsios-Rentzos [1]
University of the Aegean
Nielce Meneguelo Lobo da Costa [2]
Anhanguera University of São Paulo
Maria Elisabette Brisola Brito Prado [3]
Anhanguera University of São Paulo
Francois Kalavasis [4]
University of the Aegean

RESUMO

Em todos os sistemas educacionais hierárquicos são importantes os pontos de vista dos diretores de escola, pois eles afetam crucialmente as práticas de sala de aula e as relações entre todos os protagonistas da unidade escolar. Neste trabalho, adotamos aspectos da abordagem para a teoria dos sistemas, com o objetivo de investigar os pontos de vista dos diretores em serviço sobre a matemática, tanto como uma disciplina curricular como as práticas gerenciais do sistema de ensino, buscando entender as interações entre seus pontos de vista em relação à matemática e à organização dessa disciplina no sistema. A Matemática no contexto escolar foi investigada considerando três focos: as representações simbólicas/ regulamentares, as representações pragmáticas e as intencionalidades das ações adotadas. Um estudo comparativo qualitativo foi realizado em pequena escala (N = 59), incluindo diretores de escolas públicas em serviço em uma região de São Paulo, no Brasil (NB = 29) e diretores de escolas da Grécia (NG = 30). Os resultados das comparações intraculturais e interculturais sugerem convergências e divergências noemáticas entre

[1] amoutsiosrentzos@aegean.gr
[2] nielce.lobo@gmail.com
[3] bette.prado@gmail.com
[4] kalabas@aegean.gr

os pontos de vista epistemológicos adotados sobre a matemática pelos diretores gregos e pelos seus colegas brasileiros, também evidenciam pontos de vista sobre a matemática como um curso no ano letivo, revelando então interações intra e intersistêmicas.

Palavras-Chave: Sistemas. Pontos de vista. Crenças. Unidade escolar. Diretores escolares.

ABSTRACT

In all hierarchical educational systems, the importance of the views of the school principals crucially affect the classroom practices and relationships amongst all the protagonists in the school unit. In this paper we adopt a soft systems theory approach, in order to investigate the views that in-service school principals hold about mathematics, both as a discipline within the system of all disciplines and as a course within the system of courses taught in school, thus attempting to comprehend the interactions between their views concerning mathematics and their organising mathematics as a course. Mathematics as a school course was investigated through a tri-focussed approach contrasting the symbolic/normative, the pragmatic representations and the desired/intentioned actions. A small scale (N=59) quantitative comparative study was conducted, including in-service school principals from Brazil (N_B=29) and Greece (N_G=30). The results of the conducted intra-cultural and inter-cultural comparisons suggest noematic convergences and divergences in the epistemic views about mathematics held by the Greek principals and their Brazilian counterparts, which are also evident in their views about mathematics as a school course, thus revealing intra- and inter- systemic interactions.

Keywords: System. Views. Beliefs. School unit. School principals.

1MATHEMATICS THROUGH A SYSTEMS THEORY PERSPECTIVE

The notion of 'relationships' seems to be embedded within the very essence of mathematical thinking, since mathematics thinking occurs in the relationship amongst different representations of a notion, not being entrapped in a single representation (Duval 2006). In accordance with this perspective, we draw upon a *systems theory* perspective (Bertalanfy, 1975) to describe the complex phenomena with respect to mathematics and the school unit. A *system* is a set of interacting and interrelating parts forming an integrated whole, which has a specific purpose (Bertalanfy, 1975). In mathematics education research, systemic ideas have been discussed in relation with, amongst others, mathematics teaching and learning (Cobb & Jackson, 2008; Davis & Simmt, 2003; English, 2007, 2008; Kalavasis, Kafoussi, Skoumpourdi & Tatsis, 2010; Wittmann, 2001) and curricular reform (Begg, 2003, 2005; Bouvier, Boisclair, Gagnon, Kazadi & Samson, 2010; English 2007, 2008).

In previous studies (Moutsios-Rentzos & Kalavasis, 2012, 2013), we presented a research framework (theoretical and methodological) aiming to investigate the educational protagonists' views about mathematics and their corresponding practices. In line with this perspective, Moutsios-Rentzos, Kalavasis and Vlachos (2012) were interested in identifying, comparing and contrasting the views that the school principals hold about mathematics and their managerial practices when mathematics is considered as an element of the system of all disciplines and of the school system. Note that this perspective enables a qualitative shift on the type of questions we seek to answer: for example, instead of investigating "Is mathematics a useful subject?", we investigate "Is mathematics *more* useful than *other* disciplines?".

In the present study, we draw upon this framework to compare and contrast the views and practices concerning mathematics of the school principals of two countries: Brazil and Greece. In both countries the value of mathematics is greatly appreciated as shown by the hours of teaching dedicated in the curricula. For example, the hours allocated for mathematics teaching in Basic Education in Brazil is outnumbered only by the hours allocated for the teaching of Portuguese. Furthermore, mathematics is one of the courses that the students are required to know in order to enter the majority of the university departments (including non-immediately 'relevant' studies, such as

humanities). The situation in Greece is similar with respect to both the sheer volume of the curriculum hours dedicated to mathematics and about the importance of mathematics success in order to enter university.

Nevertheless, the importance of mathematics in both educational systems appears not to be matched by the performance of both countries in the PISA assessment (both are placed in the lower end of the PISA rankings), gathering the concern of policy makers in both countries. In Greece, this is depicted in the numerous attempts for educational reform during the last decades, with a special focus on the last two years of high-school, linked with the way of entering university. In Brazil, the focus appears to be on the teachers' professional development and related activities, as it posited that the "continuing teacher education is favoured by policies and training activities are developed through workshops, lectures, short courses and long duration, presence ones and by distance education" (Gatti, Barreto & André, 2011, p. 264).

The policy makers of both countries appear to have realised that the current mathematics education in-school practices do not produce the expected outcomes, thus rendering crucial their taking appropriate 'corrective' actions. Recently in the State of São Paulo (Brazil), there has been a move from a relatively decentralised system for Basic Education (6-17 years old) based on the National Curriculum Parameters (PCN) to a centralised system (the Official Curriculum of the State of Sao Paulo), including, for example, the everyday use of "Teacher Notebook" and "Student Notebook" provided by the central secretary of education of this State (SEESP). This movement radically differs from the situation in Greece, where the educational system is currently transformed from an absolutely centralised system towards a decentralised system consisting of semi-autonomous school units that set their own pedagogical targets adapted according to the conditions within which each unit is defined. Thus, it appears that both educational systems are in a dynamic state, in the process of reaching a new equilibrium.

Considering the educational and the socio-economic commonalities and differences of both countries, we argue that a 'snapshot' of the school principals' views about mathematics in such a dynamic state will help in revealing those aspects of the phenomenon that are inter-cultural and those linked to the special characteristics of each country. Consequently, the purpose of this study is to investigate whether or not

the sociocultural, economic and structural differences (the model of the emerging economy society vs. the European model) are somehow evident in the identified views and practices of the in-service principals, thus offering deeper insights on the inter-cultural and intra-cultural aspects of the principals' constructions about mathematics.

A CO-DEVELOPED THEORETICAL-METHODOLOGICAL APPROACH

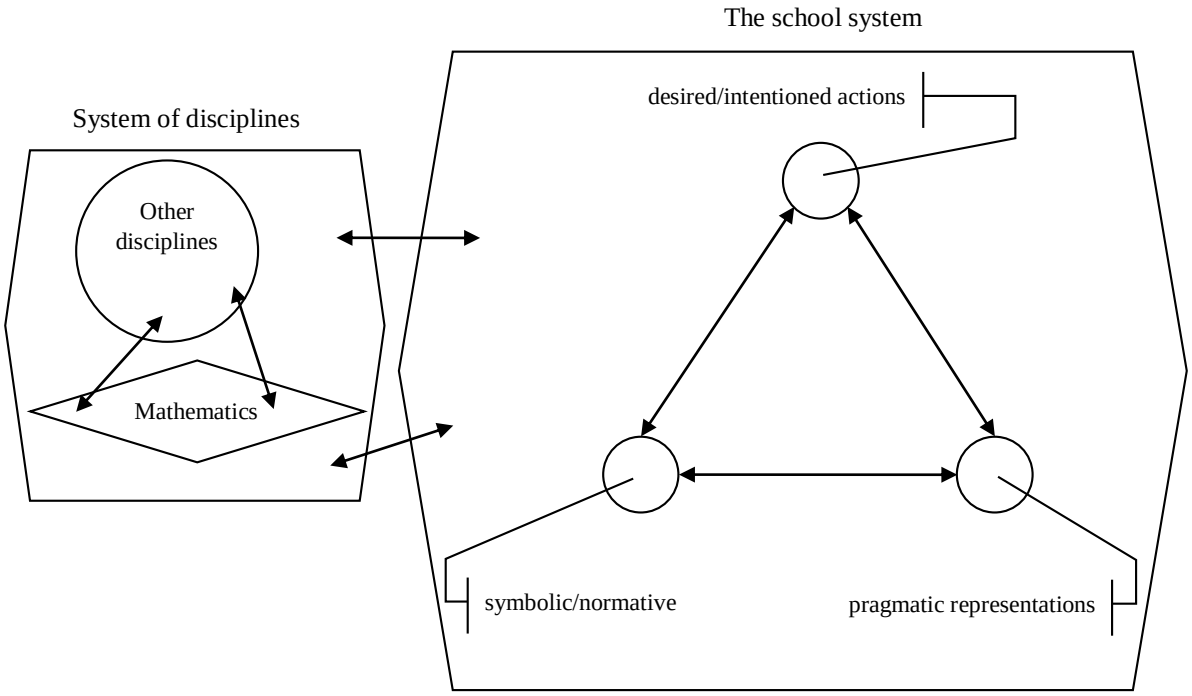
The theoretical framework

Moutsios-Rentzos and Kalavasis (2012, 2013) introduced a co-developed theoretical-methodological research framework in order to investigate the views that the educational protagonists (teachers, principals, students, parents etc) hold about mathematics. The protagonists' views are considered in two interconnected systems: a) the *system of all disciplines*, and b) the *school system*. The former investigates the view of mathematics in comparison with the other disciplines. This conceptual shift allows the identification of the 'special' characteristics of mathematics within the system of all disciplines, providing a systemic, relational view, rather than an isolated, singular view. Furthermore, regarding the school system, each of the protagonists is required to assume the role that (s)he plays within the school unit through a *tri-focussed* perspective: a) the *symbolic/normative* (the perceived official regulations), b) the *pragmatic representations* (the perceived current state of school practices), and the *desired/intentioned actions* (the personal hypothetical actions, assuming the power to implement them). Each focus essentially constitutes a sub-system as it is affected by the broader constructions from which the focus of each perspective derives (*cf* 'belief systems', for a related, yet qualitatively distinct, notion; Green, 1971). For example, the desired/intentioned actions focus about mathematics in the school unit is settled within the broader system of the desired/intentioned actions that constitute the individual's desired/intentioned construction, whilst the pragmatic representations focus is affected by the broader pragmatic representations that an individual may hold.

The approach is diagrammatically outlined in Figure 1, where the hexagons represent the two systems, the circles sub-systems of a system, the rhombi represent

elements of a system and the left right arrows represent inter-systemic and intra-systemic interactions.

Figure 1: An inter-systemic, multi-focussed approach to views about mathematics.



Overall, for the purposes of the present study, we adopt this inter-systemic, multi-focussed approach to the investigation about the views that the school principals hold about mathematics.

The instrument

The aforementioned approach has been operationalised through a three-part questionnaire (Moutsios-Rentzos & Kalavasis, 2012, 2013, submitted), which was adapted for the purposes of this study. The structure and content of the research instrument employed in this study is outlined in this section.

The first part of the questionnaire includes 11 closed items investigating the principals' epistemic views about mathematics within the system of all disciplines, focussing on five topics: *usefulness* (three items), *importance* (two items), *reasoning* (two items), *epistemology* (three items), and *truth* (one item). The topic of each item is expressed in way that explicitly asks the participants to provide an answer that

compares mathematics with the other disciplines. For example, “Do you think that mathematics is more useful in comparison with other disciplines?”. Moreover, the participants’ answers are two-faceted: first, a ‘Yes/No’ dichotomy indicating their agreement or disagreement, followed by a 4-point Likert scale to identify their degree of agreement or disagreement. In this way, we intend to first obtain a positive or negative reaction and then the degree of this reaction (which psychologically is not feasible with a simple Likert item). In Figure 2, a sample item of the first part of the questionnaire is presented.

Figure 2: A sample of the triplets of items included in the questionnaire.

Do you think that mathematics more than other disciplines promotes the development of logical reasoning?	Y	1	2	3	4
	N	1	2	3	4

The second part of the questionnaire focuses on the principal’s views about mathematics as a course within the system of school courses, in line with our tri-focussed approach (‘symbolic/normative’ – ‘pragmatic representations’ – ‘desired/intentioned actions’). Thirteen topics are investigated through 13 triplets of closed items. The topics investigated were in line with *seven strands*: *school time* (three triplets), *budget* (one triplet), *class arrangement* (one triplet), *didactics* (three triplets), *professional development* (one triplet), *assessment* (one triplet), *cooperation* (one triplet), *school success* (one triplet). Each triplet has a common part (the topic) and a unique part (in accordance with each of the three foci) inquiring both the agreement/disagreement and the degree of agreement/disagreement (following the same format as in the first part of the questionnaire). In Figure 3, a sample triplet is presented.

Figure 3: A sample of the triplets of items included in the second part of the questionnaire

According to your opinion, should the official regulations allocate a bigger part of the budget in materials for the teaching of mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4
Do you think that in reality in schools a bigger part of the budget is spent in materials for teaching mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4
As a school principal and assuming you had the power, would you allocate a bigger part of the budget in materials for teaching mathematics in relation with other courses?	Y	1	2	3	4
	N	1	2	3	4

The last part of the questionnaire consists of questions investigating the participants' age, education, working experience (teaching and administrative) and professional development.

METHODOLOGY

Sample

Fifty nine in-service principals working in Brazil and Greece participated in the study (N=59). Considering the Brazilian sample, twenty nine school principals working in *Basic Education* in an economically developed region of São Paulo (DE Norte 2), a large Brazilian city were included in the study, representing the totality of the schools of this region (n_B=29). Basic Education in Brazil is divided into *Ensino Fundamental I* (6-10 years old), *Ensino Fundamental II* (11-14 years old) and *Ensino Médio* (15-17 years old). In Ensino Fundamental II the students are taught mathematics 5 hours per week and in Ensino Médio mathematics is taught for 4 hours per week.

Regarding Greece, for these year groups, education is divided in Dimotiko (6-12 years old), Gymnasio (13-15 years old) and Lykeio (16-18 years old). In Dimotiko and Gymansio the students are taught mathematics 4 hours per week, whereas in the first two grades of Lykeio mathematics is taught for 4 to 5 hours per week. In the third grade of Lykeio there is a range from 2 to 7 hours depending on the specialisation chosen by the students. Bearing in mind that in Brazil a school principal may work in a school offering more than one level of Basic Education (which is not possible in Greece), we included in the study thirty in-service principals working in specific region a large Greek city in schools of all three levels (n_G=30).

It should be noted that though the two samples are relatively small, we argue that they are sufficient for the purposes of the present study: to reveal inter-systemic and intra-systemic convergences and divergences. By focussing on only one Brazilian region and on only one Greek region, we attempt to minimise the socio-cultural diversity that exists within different regions of large cities, thus allowing for our intended

investigations in the principals' constructions about mathematics of these specific regions.

The characteristics of the participants of the study are outlined in Table 1.

Table 1: The participants of the study

		Brazil		Greece		Brazil vs. Greece[g]
Gender	Male	5[a]	20.8%[b]	18	60.0%	***P*=0.006**[d]
	Female	19	79.2%	12	40.0%	
Age	<34	1	4.0%	0	0.0%	U=354.5, z=-0.420, P=0.669[e]
	34-45	7	28.0%	6	20.0%	
	46-57	14	56.0%	23	76.7%	
	>57	3	12.0%	1	3.3%	
Professional development						
administration	No	11	42.3%	17	58.6%	P=0.285[d]
	Yes	15	57.7%	12	41.4%	
didactics	No	7	26.9%	6	20.7%	P=0.752[d]
	Yes	19	73.1%	23	79.3%	
Postgraduate degree	No	13	50.0%	20	69.0%	P=0.178[d]
	Yes	13	50.0%	9	31.0%	
School experience	27(11) [12,67], 24[c]			30(10) [13,51], 29		t(54)=-1.281, ***P*=0.206**
administrative post	12(8) [1,40], 10			7(5) [1,16], 6		t(54)=2.341, ***P*=0.023**
teaching post	15(6) [5,28], 14			23(7) [7,35], 24		t(56)=-4.797, P<0.001
current post	10(7) [1,30], 10			11(9) [1,27], 10		t(56)=-0.312, P=0.756

[a]: Frequency, [b]: Valid percent, [c]: M(SD) [min,max], Mdn, [d]: Fisher's exact test, [e]: Mann-Whitney U test, [f]: Independent samples Student's t-test, [g]: Statistically significant differences in bold.

Drawing upon the characteristics outlined in Table 1, it appears that the two samples do no statistically significantly differ with respect to age, professional development (administrative and didactical), their obtaining a postgraduate degree and their overall school experience. Nevertheless, the school experience seems to be statistically differently distributed between the two countries, with the Greek principals having spent more than two thirds of their experience in a teaching post, which means that the Brazilian principals of this sample are more experienced principals, whilst the Greek principals of this sample are more experienced teachers. Finally, the Brazilian sample appears to include statistically more female principals than the Greek sample.

Procedures

The quantitative analyses were conducted with IBM SPSS Statistics 22. The participants' answers in each item were scored as follows: for each response, we note '+1' or '-1' respectively for a 'Yes' or 'No' and subsequently we calculate the *intensity* for each participant's answer on an item as the product of '+1' or '-1' times the degree of agreement/disagreement as identified in the questionnaire ('1' to '4'). For example, if someone answered 'Yes' and '3' on an item this resulted to an intensity of '+3'. Accordingly, an answer of 'No' and '2' resulted in '-2'.

The normality of the obtained data was examined through Kolmogorov-Smirnoff tests and visual inspection of the P-P plots, suggesting the non-normality of the collected data (except for the school experience data). For the comparisons of two groups the two-tailed unpaired Student's *t* test, the Mann-Whitney *U* test and Fisher's exact test was used (depending on the variable type and on the normality of the data). In order to identify statistically intra-population significant differences amongst the three foci, Friedman's ANOVAs were conducted (followed by the Wilcoxon sign rank tests with Bonferroni correction applied), whilst for inter-population differences MANOVAs were conducted. *P* value of <0.05 was considered to be statistically significant.

RESULTS

Mathematics as a discipline: inter-cultural comparisons

The results of our analysis suggested that the Greek in-service principals statistically more than their Brazilian colleagues, viewed mathematics as being more important than other disciplines, more useful in everyday life more complex problems and that the mathematical results are less disputed amongst the scientific results (see Table 2). The latter appears to be the strongest disagreement between the two samples revealing a strong epistemological difference: the Brazilian sample appears to hold almost neutral views (M_B=-0.3), whilst the Greek principals seem to be almost certain (M_G=3.2, out of a theoretical maximum of '4').

On the other hand, both populations appear to agree that mathematics "more than other disciplines promote the development of logical reasoning" (M_B=2.3 and M_G=2.7, out of a theoretical maximum of '4') and that mathematics "is necessary for the development of all the other disciplines" (M_B=2.7 and M_G=2.4, out of a theoretical maximum of '4').

In the rest of the items, the principals hold not statistically significant different (neutral or relatively weak) views either in the same direction views or close to neutral (between '-1' and '+1').

Furthermore, it is noteworthy that both groups disagree that mathematics has a weaker relationship with the real world in comparison to other disciplines (M_B=-1.5 and M_G=-1.6, out of a theoretical minimum of '-4')and both agree at the same degree that, more than other disciplines, mathematics requires a type of thinking that is different from the usual (M_B=1.2 and M_G=1.2, out of a theoretical maximum of '4').

Table 2: Mathematics as a discipline with the system of all disciplines (inter-population comparisons)

	Brazil	Greece	U^b	Z	P
Do you agree that mathematics					
is more important in comparison with other disciplines?	-1.7[a]	1.0	238.5	-3.056	**0.002**[c]
is more useful in comparison with other disciplines?	-1.1	0.8	312.5	-1.902	0.057
more than other disciplines promote the development of logical reasoning?	2.3	2.7	411.0	-0.389	0.702
is more useful in our dealing with everyday problems in comparison with other disciplines?	-0.1	0.6	367.0	-1.043	0.301
is developed better when all disciplines flourish?	1.1	1.8	377.0	-0.900	0.374
become more useful in everyday life in comparison with other disciplines, as the complexity of the problems we deal with increases?	-0.8	0.8	302.0	-2.038	**0.041**
more than other disciplines, is mainly a symbolic language?	1.4	0.8	382.0	-0.820	0.417
more than other disciplines, requires a type of thinking that is different from the usual?	1.2	1.2	410.5	-0.381	0.708
has weaker relationship with the real world in comparison with other disciplines?	-1.5	-1.6	400.0	-0.546	0.590
in comparison with other disciplines, is the discipline the results of which are the least disputed?	-0.3	3.2	212.5	-3.524	<**0.001**
is necessary for the development of all the other disciplines?	2.7	2.4	388.0	-0.753	0.457

[a]: M, values may range from '-4' (maximum disagreement) to '+4' (maximum agreement), [b]: Mann-Whitney U test, [c]: Statistically significant differences in bold.

Mathematics as a course in the school unit

The results of the intra-populations comparisons of the views that the principals hold about mathematics as a course according to our tri-focussed approach (symbolic/normative, pragmatic representations, desired/intentioned actions) are summarised in Table 3.

Table 3: Mathematics as a course within the school unit system (intra-cultural comparisons)

		Brazil					Greece				
		M^b	χ^2(df), P	S/N	PR	D/IA	M	χ^2(df), P	S/N	PR	D/IA
a bigger part of the budget in materials for the teaching of mathematics in relation with other courses	S/N[a]	-0.9	$\chi^2(2)=3.444$	-			-0.7	$\chi^2(2)=3.444$	-	*[d,e]	*
	PR	-2.2	$P=0.187$[c]		-		-2.7	**$P<0.001$**[f]		-	ns
	D/IA	-0.8				-	-0.6				-
more class hours for the teaching of mathematics in relation with other courses	S/N	-0.3	$\chi^2(2)=5.154$	-			-0.6	$\chi^2(2)=4.753$	-		
	PR	0.1	$P=0.076$		-		-0.6	$P=0.093$		-	
	D/IA	-1.6				-	0.2				-
special arrangement of desks in class for the mathematics hour, different from the arrangement in other courses	S/N	-1.2	$\chi^2(2)=10.949$	-	ns	*	-1.3	$\chi^2(2)=10.073$	-	ns	*
	PR	-1.3	**$P=0.004$**		-	*	-2.9	**$P=0.005$**		-	*
	D/IA	0.8				-	-1.2				-
the teaching of mathematics in the morning school hours more often than other courses	S/N	-1.0	$\chi^2(2)=21.593$	-	*	ns	1.9	$\chi^2(2)=2.822$	-		
	PR	-2.6	**$P<0.001$**		-	*	1.6	$P=0.253$		-	
	D/IA	-0.0				-	2.7				-
for the students that face difficulties in mathematics, mathematics taught jointly with other courses	S/N	2.8	$\chi^2(2)=6.703$	-	ns	ns	1.4	$\chi^2(2)=2.607$	-		
	PR	2.4	**$P=0.040$**		-	*	1.9	$P=0.292$		-	
	D/IA	3.3				-	1.0				-
mathematics more than other courses taught as much as possible in consecutive class hours	S/N	1.9	$\chi^2(2)=5.944$	-			0.0	$\chi^2(2)=1.361$	-		
	PR	0.3	$P=0.051$		-		-0.8	$P=0.529$		-	
	D/IA	1.5				-	0.2				-
use of materials in the teaching of mathematics more than other courses	S/N	-0.9	$\chi^2(2)=7.165$	-	ns	ns	0.0	$\chi^2(2)=11.227$	-	*	ns
	PR	-2.4	**$P=0.026$**		-	*	-1.8	**$P=0.004$**		-	*
	D/IA	-0.4				-	-0.3				-
special way of assessing mathematics in relation with other courses	S/N	0.5	$\chi^2(2)=7.400$	-	ns	ns	1.2	$\chi^2(2)=11.237$	-	*	ns
	PR	-0.9	**$P=0.025$**		-	*	-0.8	**$P=0.003$**		-	ns
	D/IA	1.0				-	0.8				-
more training programmes offered about the teaching of mathematics than about the teaching of other courses	S/N	2.6	$\chi^2(2)=24.964$	-	*	ns	-1.0	$\chi^2(2)=9.108$	-	ns	*
	PR	-1.7	**$P<0.001$**		-	*	-1.4	**$P=0.009$**		-	*
	D/IA	1.9				-	1.0				-
use of ICT in the teaching of mathematics more than other courses	S/N	1.3	$\chi^2(2)=18.960$	-	*	ns	1.1	$\chi^2(2)=19.662$	-	*	ns
	PR	-2.0	**$P<0.001$**		-	*	-1.5	**$P<0.001$**		-	ns
	D/IA	0.8				-	-0.1				-
	S/N	-0.0	$\chi^2(2)=12.781$	-	*	ns	0.7	$\chi^2(2)=7.000$	-	*	ns

cooperation with the families about mathematics more than other courses	PR	-2.7	***P*=0.001**	-	*	-1.5	***P*=0.029**	-	*
	D/IA	-0.6			-	0.3			-
the success of a school unit linked with the success in mathematics more than with the success in other courses	S/N	-2.6	$\chi^2(2)=1.550$	-		-1.4	$\chi^2(2)=0.824$	-	
	PR	-2.2	$P=0.481$	-		-1.2	$P=0.752$	-	
	D/IA	-2.2			-	-1.5			-

[a]: Symbolic/Normative, Pragmatic Representations, Desired/Intentioned Actions, [b]: values may range from '-4' (maximum disagreement) to '+4' (maximum agreement), [c]: Friedman's ANOVA, [d]: Post-hoc contrasts via Wilcoxon sign rank tests with Bonferroni correction applied, [e]: Statistically significant post-hoc contrasts with '*' and not statistically significant post-hoc contrasts with 'ns', [f]: Statistically differences in bold.

The above can be *qualitatively* classified as follows:

1. Negative pragmatic representations and symbolic/normative *combined with* even stronger negative desired/intentioned actions
   - special arrangement of desks in class for the mathematics hour, different from the arrangement in other courses (Greece)
2. Negative pragmatic representations and symbolic/normative *contrasting* neutral desired/intentioned actions
   - a bigger part of the budget in materials for the teaching of mathematics in relation with other courses (Greece)
   - special arrangement of desks in class for the mathematics hour, different from the arrangement in other courses (Brazil)
   - the teaching of mathematics in the morning school hours more often than other courses (Brazil)
   - use of materials in the teaching of mathematics more than other courses (Brazil)
3. Negative pragmatic representations and symbolic/normative *contrasting* positive desired/intentioned actions
   - more training programmes offered about the teaching of mathematics than about the teaching of other courses (Greece)
4. Negative pragmatic representations *contrasting* neutral desired/intentioned actions and symbolic/normative

- use of materials in the teaching of mathematics more than other courses (Greece)
- cooperation with the families about mathematics more than other courses (Brazil & Greece)

5. Negative pragmatic representations *contrasting* positive desired/intentioned actions and symbolic/normative
   - special way of assessing mathematics in relation with other courses (Brazil & Greece)
   - more training programmes offered about the teaching of mathematics than about the teaching of other courses (Brazil)
   - use of ICT in the teaching of mathematics more than other courses (Brazil & Greece)
6. Positive pragmatic representations and symbolic/normative combined with even stronger positive desired/intentioned actions
   - for the students that face difficulties in mathematics, mathematics taught jointly with other courses (Brazil)

It should be noted that the identified statistically different systemic intra-population constructions were linked with negative pragmatic representations, with only one positive and, notably, with no neutral. Moreover, only three topics were qualitatively similar in both countries. Furthermore, all the identified statistically differences can be grouped in two broad categories: a) two foci in the same direction (positive, neutral, negative) with potentially varied intensity and one focus in a different direction, and b) the three foci in the same direction with different intensity.

Regarding the inter-cultural comparisons of the views that the principals hold about mathematics as a course according to our tri-focussed approach (symbolic/normative, pragmatic representations, desired/intentioned actions), the results of the analyses are summarised in Table 4.

Table 4: Mathematics as a course within the school unit system (inter-cultural comparisons)

	M_B[b]	M_G[b]	F(*df*, error *df*)[c]	*P*[d]
S/N[a]	-0.9	-0.7	F(3,55)=0.545	0.654

a bigger part of the budget in materials for the teaching of mathematics in relation with other courses	PR	-2.2	-2.7		
	D/IA	-0.8	-0.6		
more class hours for the teaching of mathematics in relation with other courses	S/N	-0.3	-0.6	F(3,55)=1.161	**0.006**
	PR	0.1	-0.6		
	D/IA	-1.6	0.2		
special arrangement of desks in class for the mathematics hour, different from the arrangement in other courses	S/N	-1.2	-1.3	F(3,55)=6.144	**0.001**
	PR	-1.3	-2.9		
	D/IA	0.8	-1.2		
the teaching of mathematics in the morning school hours more often than other courses	S/N	-1.0	1.9	F(3,55)=19.879	<**0.001**
	PR	-2.6	1.6		
	D/IA	0.0	2.7		
for the students that face difficulties in mathematics, mathematics taught jointly with other courses	S/N	2.8	1.4	F(3,55)=7.193	<**0.001**
	PR	2.4	1.9		
	D/IA	3.3	1.0		
mathematics more than other courses taught as much as possible in consecutive class hours	S/N	1.9	0.0	F(3,55)=2.130	0.107
	PR	0.3	-0.8		
	D/IA	1.5	0.2		
use of materials in the teaching of mathematics more than other courses	S/N	-0.9	0.0	F(3,55)=1.042	0.381
	PR	-2.4	-1.8		
	D/IA	-0.4	-0.3		
special way of assessing mathematics in relation with other courses	S/N	0.5	1.2	F(3,55)=0.676	0.570
	PR	-0.9	-0.8		
	D/IA	1.0	0.8		
more training programmes offered about the teaching of mathematics than about the teaching of other courses	S/N	2.6	-1.0	F(3,55)=9.079	<**0.001**
	PR	-1.7	-1.4		
	D/IA	1.9	1.0		
use of ICT in the teaching of mathematics more than other courses	S/N	1.3	1.1	F(3,55)=0.741	0.532
	PR	-2.0	-1.5		
	D/IA	0.8	-0.1		
cooperation with the families about mathematics more than other courses	S/N	-0.0	0.7	F(3,55)=2.024	0.121
	PR	-2.7	-1.5		
	D/IA	-0.6	0.3		
the success of a school unit linked with the success in mathematics more than with the success in other courses	S/N	-2.6	-1.4	F(3,55)=1.397	0.253
	PR	-2.2	-1.2		
	D/IA	-2.2	-1.5		

[a]: Symbolic/Normative, Pragmatic Representations, Desired/Intentioned Actions, [b]: values may range from '-4' (maximum disagreement) to '+4' (maximum agreement), [c]: MANOVA, [d]: Statistically differences in bold

In line with the intra-cultural analyses, the identified statistically differences in the tri-focussed constructions between the Brazilian principals and the Greek principals appear to be qualitatively classified as follows:

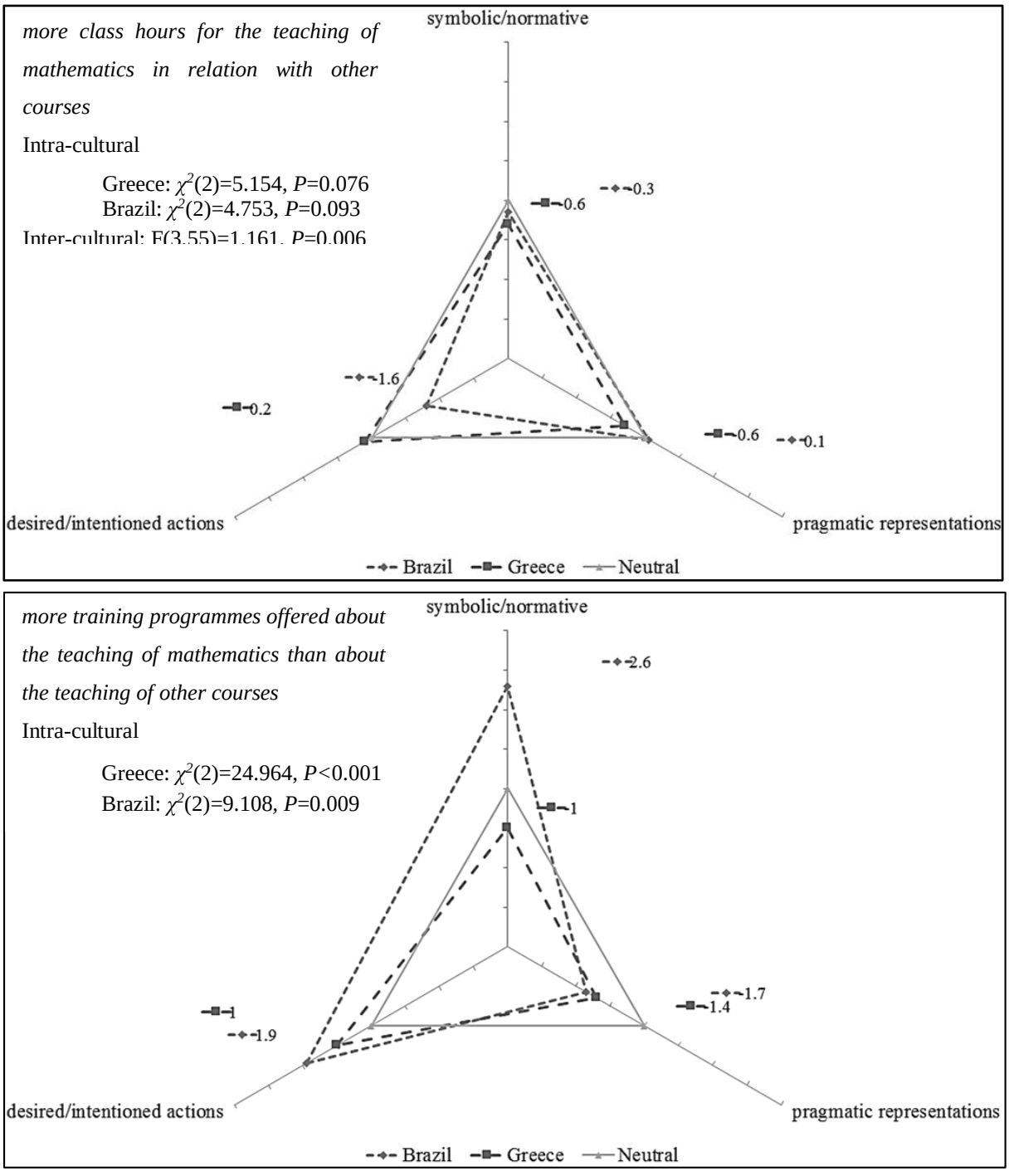
1. Two foci in the same direction (positive, neutral, negative) with potentially varied intensity and one focus in a different direction
    - more class hours for the teaching of mathematics in relation with other courses
    - special arrangement of desks in class for the mathematics hour, different from the arrangement in other courses
    - more training programmes offered about the teaching of mathematics than about the teaching of other courses
2. The three foci in the same direction with different intensity for the two countries
    - for the students that face difficulties in mathematics, mathematics taught jointly with other courses
3. The three foci in different directions for each country
    - the teaching of mathematics in the morning school hours more often than other courses

Furthermore, in order to obtain an overview of the inter-foci comparisons (both the intra-cultural and the inter-cultural comparisons), the results can be visually represented in a comprehensive multi-levelled diagram constructed for this purpose (for detailed discussion about the multifaceted purposes and utilities of this type of diagram, see Moutsios-Rentzos & Kalavasis, submitted).

In Figure 4, two examples of such diagrams are presented representing the results for the topic "more class hours for the teaching of mathematics in relation with other courses" and for the topic "more training programmes offered about the teaching of mathematics than about the teaching of other courses". Notice, that even though the two topics are qualitatively classified in the same category (only one focus in different direction), the diagram allows to grasp the qualitative difference between the two systemic views taking into consideration the 'neutral' view. Thus, for the first topic the neutral views are contrasted with the negative desired/intentioned actions professed by the Brazilian principals. On the other hand, for the topic "more training programmes offered about the teaching of mathematics than about the teaching of other courses",

the Brazilian and the Greek systemic constructions are skewed both with respect to each other and to the neutral.

Figure 4: Mathematics as a course within the school unit system (both inter- and intra-cultural comparisons).



5. DISCUSSION

The investigations within and across the system of all disciplines and the school unit system revealed convergences and divergences in the principals' constructions. Considering the system of all disciplines, the principals of both countries appear to consider mathematics as having special characteristics when compared with the other disciplines, in terms of three aspects: *reasoning* (mathematics 'promotes the development of logical reasoning'; mathematics 'is mainly a symbolic language'; mathematics 'requires a type of thinking that is different from the usual'), *epistemic value* (mathematics 'is necessary for the development of all the other disciplines'; mathematics 'is developed better when all disciplines flourish'), *pragmatic value* (mathematics does *not* have 'a weaker relationship with the real world in comparison with other disciplines'). Thus, there appears to be a noematic convergence with respect to the fact that mathematics is special when compared with the other disciplines in all three aspects (reasoning, epistemic value, pragmatic value). Nevertheless, this convergence seems to be qualitatively differentiated with the Greek principals statistically significantly differing from the Brazilian principals. The Greek principals view mathematics as being 'more important in comparison with other disciplines' and they note that mathematics 'is the discipline the results of which are the least disputed' (epistemic value divergences), whilst they note that mathematics is 'more useful in everyday life in comparison with other disciplines, as the complexity of the problems we deal with increases' (pragmatic value divergence). Notice that no divergences are identified with respect to the special characteristics of mathematical reasoning. It seems that there is an inter-cultural convergence about the special epistemic status of mathematics, which –echoing the Ernest's (1991) conceptualisations of absolutist, fallibilist and utilitarian– appears to be experienced in qualitatively different 'scents': the Greek principals' mathematics-is-special blend is scented with absolutistic and utilitarian 'aromas' (slightly elitistic, since mathematics is more useful in complexity), whilst the Brazilian principals' mathematics-is-special blend is scented mainly with fallibilistic 'aromas'.

Considering the school unit systems, the tri-focused intra-cultural and inter-cultural investigations revealed a complex experienced reality. In general, both the Greek and the Brazilian principals appear in most cases not to experience the mathematics school course as being special in terms of the pragmatic representations and of their desired/intentioned actions. Nevertheless, concentrating in the intra-cultural inter-foci comparisons, it seems that the mathematics school course is considered to be special in terms of the way that both the Greek and the Brazilian principals would *intend to act* assuming their having the power about: *assessment* ('special way of assessing mathematics in relation with other courses'), *didactics* ('use of ICT in the teaching of mathematics more than other courses'; 'for the students that face difficulties in mathematics, mathematics taught jointly with other courses'), and *professional development* ('more training programmes offered about the teaching of mathematics than about the teaching of other courses'). On the other hand, concentrating in the inter-cultural comparisons of the tri-focussed experienced reality, statistically significant divergences in the principals' constructions about their were identified in: *school time* (the Brazilians' negative desired/intentioned actions for allocating more class hours to mathematics in comparison with the Greeks' neutral intentions; the Greeks' positive desired/intentioned actions for more morning school hours in comparison with the Brazilians' neutral intentions), *class arrangement* (the Brazilians' positive desired/intentioned actions for different class arrangement in mathematics contrasting the Greeks' negative intentions), and *professional development* (the Brazilians' positive symbolic/normative construction for more training programmes about mathematics contrasting the Greeks' negative construction).

Consequently, in contrast with the professed epistemic views, it seems that in general the mathematics school course is not experienced in everyday reality as being special. Nevertheless, it is posited that the principals' desired/intentioned actions appear to qualitatively differ in ways that accord with their professed epistemic views. The Greek principals would favour the mathematics being taught in the morning class hours, being at the same time neutral about more school hours of teaching mathematics. These intentions reflect their epistemic views as mathematics being more important than the other disciplines, as in the Greek school reality the morning school hours are considered to be the important ones: the students are not tired form teaching and importantly the school hours are actually longer than the last hours of the

day (due to the way that the schedule is constructed). Therefore, the Greek principals appear to experience mathematics as being more important as a discipline and as a course, thus choosing to allocate more important school hours to mathematics and not just more school hours. At the same time, the Greek principals are in stark contrast with the Brazilian principals' symbolic/normative constructions for more training programmes about the teaching of mathematics. This combined with the fact that the Brazilian principals' intentions for special class arrangements in the teaching of mathematics and the Greeks' neutral stance and the fallibilistic aroma of the Brazilians' mathematics-is special scent may explain this phenomenon since it is well documented that the absolutistic stance has been linked with more traditional, teacher-centred pedagogies. It could be argued that the Brazilians' epistemic views affected their intentioned actions in opting for more radical, immediately affecting everyday pedagogies (such as different class arrangement), whilst the Greeks holding more absolutist epistemic views chose less immediate actions, affecting the structure of the whole schedule, rather than the pedagogy itself.

6. CONCLUDING REMARKS

Overall, notwithstanding the generalisability limitations of the present study due to the relatively small samples, we posit that the proposed theoretical-methodological systemic approach allowed our revealing and meaningfully relate divergences and convergences in the principals' constructions about mathematics as a discipline amongst the other disciplines and about mathematics as a school course amongst the other school courses. Importantly, the differences of epistemic perspective between the Brazilian principals and the Greek principals is also reflected in their school managerial perspective, mainly in the desired/intentioned actions. The employed systemic methodology revealed the complexity of the distinct incongruous constructions that constitute the principals' views about mathematics, supporting the hypothesis that in hierarchical educational systems, the school principals' views crucially affect not only the immediate classroom practices, but they transcend the mathematics value system, the desired mathematics status, and the whole cognitive

and affective complexity of mathematics teaching and learning. We posit that the adopted theoretical methodological approach allowed our realising this complexity and meaningfully investigating the interactions amongst and within different systems, including the disciplines, the school unit, the two countries-systems.

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Submitted: June, 2015

Accepted: October, 2015