GESTURE, CONCEPTUAL INTEGRATION AND MATHEMATICAL TALK

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The research reported here focuses on an examination of the conceptual underpinnings of two areas of mathematical thought, fractions and proof. The analysis makes use of the theoretical framework of conceptual integration, and draws on the modality of spontaneous gesture as an important data source. The question of how gestures evoke meaning is addressed within the context of two studies, one involving prospective elementary school teachers discussing fractions, and the other involving doctoral students in mathematics talking about and carrying out proofs. In both situations, gestures and their accompanying language are analyzed in terms of conceptual mappings from more basic conceptual spaces.

INTRODUCTION

An overarching goal of the research program of which this study is a part is to investigate the role of multimodality in doing and communicating mathematics. Multimodality has been defined as "the idea that communication and representation always draw on a multiplicity of semiotic modes of which language may be one" (Kress, 2001, p. 67-68). Mathematics in particular involves multiple modalities, including written symbols, oral speech and visual imagery (both internal and external). In addition to these modalities, spontaneous physical gesture is a modality that, until recently, has not received a great deal of attention in research into mathematical thinking and communication, yet it may serve as an important bridge between private, internal imagery (which can be difficult to express in words), and the formal, symbolic expression of mathematical ideas (Arzarello, 2006).

The purpose of this study was to examine the ways that spontaneous physical gesture is used in communicating about mathematical ideas and problem solving. A concurrent goal was to collect a corpus of spontaneous gestures produced within the context of mathematical talk, and to utilize the framework of embodied cognition and cognitive linguistics in order to make sense of these gestures.

THEORETICAL FRAMEWORK

The research takes place within the theoretical framework of embodied cognition (Varela, Thompson & Rosch, 1991), and utilizes the tools of cognitive linguistics and gesture studies (Fauconnier & Turner, 2002; McNeill, 1992, 2005). From the perspective of embodied cognition, mathematics is not a transcendental, formal collection of rules and patterns, but instead, a human intellectual product, socially-constructed yet both constrained and enabled by the physical capabilities and circumstances of human beings. Thus, the research reported here is concerned not with how students "acquire" knowledge of a pre-existing formal domain, but with how they utilize their own embodied capabilities to construct understandings that can be communicated within a community that shares a common biological and experiential heritage (Nuñéz, Edwards & Matos, 1999).

From the point of view of cognitive linguistics, language is primarily based on collections of unconscious mental mappings linking familiar experiences and ideas in order to create new ones. An important mechanism within this framework is conceptual integration (or blending) of mental spaces. As defined by Fauconnier and Turner, "Conceptual integration … connects input spaces, projects selectively to a blended space, and develops emergent structure" (Fauconnier & Turner, 2000, p.89). Conceptual integration can be seen as a general mechanism that encompasses more specific mappings such as conceptual metaphor; the latter have been used in the analysis of mathematical ideas ranging from arithmetic to calculus (e.g., Bazzini, 1991; Lakoff & Núñez, 2000; Núñez, Edwards & Matos, 1999; Pimm, 1981; Presmeg, 1991).

RELATED RESEARCH

Previous research on gesture and mathematics has examined a variety of mathematical tasks, ranging from learning to count (Alibali & diRusso, 1999; Graham, 1999) to communicating about differential equations (Rasmussen, Stephan & Whitehead, 2003). One of the findings of studies both within and outside of mathematics is that speech and can "package" complementary forms of information within the same discourse: linear, symbolic verbal language on the one hand, and global, instantaneous imagery on the other, and that this complementarity can be powerful in communicating and learning (Arzarello, 2006; Goldin-Meadow, 2003; Kita, 2000; McNeill, 2005). In several studies, learners were able to express their understanding of a new concept through gesture before they were able to express it in speech, and a "mismatch" or non-redundancy between the information expressed through gesture versus speech was an indicator of "readiness to learn" the new concept (Goldin-Meadow, 2003). The current research will examine specific gestures as evidence for the ways that the participants conceptualize mathematical ideas, within a setting involving interviews and simple problem solving.

METHODOLOGY

The data were collected in two sets of interviews. In the first study, 12 female undergraduates were interviewed in pairs for about 30 minutes about fractions, a topic that can be problematic for both children and adults. The students described how they learned fractions, and how they would define and introduce this topic to children. They also worked together to solve a set of five simple arithmetic problems involving fractions. A report on the preliminary analysis of the fractions data can be found in Edwards, 2003.

In a second study, twelve doctoral students in mathematics were also interviewed in pairs, this time for about 90 minutes. The focus of the interview was mathematical proof; students were first asked about their mathematical specializations, and then about their experiences teaching proof, whether they could categorize different types of proofs, and what they personally found difficult about proof. The students were then presented with a conjecture and worked together to create a proof for it, and, finally, judged whether a visual argument constituted a proof in their opinion.

The interviews were videotaped, and the gestures isolated and identified. The gestures were initially classified using a scheme established by psychologist David McNeill. Three of the types, or dimensions, distinguished by McNeill are: *iconic gestures*, which “bear a close formal relationship to the semantic content of speech” (in other words, which visually resemble their concrete referents); *metaphoric gestures*, where “the pictorial content presents an abstract idea rather than a concrete object or event” (McNeill, 1992, p.14); and *deixis*, a “pointing movement [that] selects a part of the gesture space” (op. cit., p. 80). These three dimensions of gesture were the most salient in the data described here.

RESULTS

The analysis presented here will focus on how we are able to “read” the meaning of a gesture, whether we are participants in the discourse or researchers trying to analyze it. Interpreting the meaning of a gesture has been called “an intuitive inferential process” (Parrill & Sweetser, 2004, p. 197), yet the analytic framework of cognitive linguistics provides guidance for this process, through the use of metaphor and conceptual integration in analyzing specific and typical gestures.

Example 1: Conceptualization of fractions

The first specific question to be addressed in the analysis of the first set of interviews is, "What information do the students' gestures (and language) offer about their conceptualization of fractions?" A basic, initial conceptualization of a fraction is framed around a part-whole model (a fraction is a part of a whole that has been divided into equal size parts). This "whole" might be a continuous area (e.g., a circle or rectangle), a collection of discrete objects (e.g., a set of children in a classroom), or a length or distance (e.g., metaphorically, the "length" of an 8-hour work day). These models correspond to four grounding metaphors for arithmetic identified by Lakoff & Núñez; specifically: object construction, object collection, measuring stick, and motion along a path (the latter two metaphors would correspond to the "part of a length or distance" model of fractions, depending on context; Lakoff & Núñez, 2000).

Both the words and the gestures utilized by the students when talking about fractions provide evidence about which unconscious metaphor underlies their understanding of this concept. When the students were asked to give a definition of fraction, only two of the twelve utilized gestures (perhaps because, rather than naturalistically explaining about fractions, this request made them feel that they had to retrieve the correct formal, verbal definition). The gestures used by these two students are described in Table 1 (abbreviations for the gesture descriptions are: RH, LH, BH= Right hand, Left hand, Both hands; C-, L- and S-shapes=ASL hand shapes).

ho	**Speech**	**Gesture Description**
G	But it's only a piece of -	LH, L-shape, cutting motion, palm toward face
G	a piece of the wh-	LH, open L, parallel to table
G	a piece of whatever we're dealing with that's whole	BH, symmetric open L-shapes, thumbs up, palms facing body
G	it's just a portion of	LH toward body, slightly curled S-shape, bounced toward body
T	a portion of a pie	slide LH fingers along edge of table

Table 1: Gestures associated with definitions of fractions

The verbal definitions given by students who did not use gestures were quite similar to the accompanying speech above, and included the following:

"I would probably put like a part of a whole."

"A part to a whole number"

"A fraction is something that breaks up whole numbers"

"You're just taking something out of the whole"

Even though there were few gestures associated with the verbal definition of a fraction, gestures for "cutting," "slicing" or "splitting" were well represented in the corpus. There were thirteen instances of such gestures, out of a corpus of 251 gestures in all. These "cutting" gestures comprised 22% of the iconic gestures and nearly 5% of the entire corpus. This suggests that the physical act of dividing something was an important component of the participants' conceptualizations of fractions.

The cutting and slicing gestures, as well as the verbal definitions referring to "parts", "breaking up" and "taking something" out of wholes constitute evidence that the students were utilizing an *object construction* metaphor for understanding fractions. Within this metaphor, numbers are objects that are composed of other objects (themselves numbers); for example, within the context of whole numbers, students with this metaphor would be able to see 5 as being "composed of" 2 plus 3 (Lakoff & Núñez, 2000).

In terms of fractions, the students' definitions referring to "breaking up" whole numbers, and "parts of wholes" suggest that the same metaphor is at work. Only a number that is constructed of parts can be split into equal sized part, i.e., fractions. More specifically, within the "Arithmetic is Object Construction" metaphor, numbers are seen as objects, with the smallest whole object corresponding to the number one (the unit). A simple or unit fraction is understood as being "a part of a unit object (made by splitting a unit into *n* parts)" and a complex fraction (*m/n*) as "an object made by fitting together *m* parts of size 1/*n*" (Lakoff & Núñez, 2000, p. 67).

This metaphor or conceptual mapping is consistent with the language and gestures used by the students in the current study. It should be noted none of the students' comment or gestures indicated an understanding of fractions in terms of object collections (i.e., a "part" of a set of discrete objects) or portions of a measuring stick or of a motion along a path. Thus, based on the data collected from these students, the source domain underlying their ideas about fractions is the idea of a number as an object constructed out of parts, an a fraction as one of those parts.

An iconic gesture for "cutting"

An example of a "cutting" gesture is shown in Figure 1, where student LR is talking about how her teacher would demonstrate fractions by cutting a pie (which may have been a real pie, or perhaps a model or manipulative). She first displays a "cross-cutting" gesture sequence (in Figure 1a) which clearly displays the process of dividing an imaginary pie or circle into halves (using a her right hand to cut perpendicularly to herself), and then fourths (a second cut at right angles to the first, not shown). During the second gesture sequence (Figure 1b), her "cutting" hand movement is similar, but this time she turns her hand clockwise only 45°, to make an "eighth" slice, and then turns counter clockwise 90° to show a second "one-eighth" slice on the left side of the "pie."

a. "...like cutting the **pie in** like <u>// pieces</u>…"

b."…and then she cut in **like eighths**"

Figure 1. An iconic gesture for "cutting"
(**bold** text indicates synchronization of speech with gesture).

Given a shared cultural background with the speaker, it is easy to interpret this gesture as referring to the action of carefully cutting an imaginary pie with a knife or similar implement. Yet it bears asking: *how do we make this interpretation?* The description of iconic gestures as those that resemble their referent begs the question of how we are able to "see" this resemblance. A hand would not be mistaken for a knife in ordinary circumstances; it is both the intentional movement and configuration of the hand, as well as the concurrent speech, that allows us to make what seems like a simple interpretation of this gesture.

From the perspective of cognitive linguistics and gesture studies, this interpretation occurs through a blend of two mental or conceptual spaces: one containing our knowledge and control of our hands and arms, and one containing our conceptualization of the act of cutting. Our knowledge of our bodies and the physical space around us has been labeled “Real Space” within gesture studies (Parrill & Sweetser, 2004). Figure 2 illustrate the conceptual blend that gives rise to the cutting gesture. The two input spaces are shown on the left and right sides of the diagram. Above, the “generic space” refers to elements that the two spaces have in common; these commonalities allow our minds to construct the blend, shown in the bottom circle. In this case, the generic space includes such features as the perpendicularity of both the hand and the knife to the surface of the table, the fact that both are narrow relative to their lengths, and that both can be moved up and down. In utilizing the affordances of her hand and arm to highlight these commonalities, LR evokes a conceptual blend that allows an interlocutor to “see” her hand as a knife being used to cut or slice something.

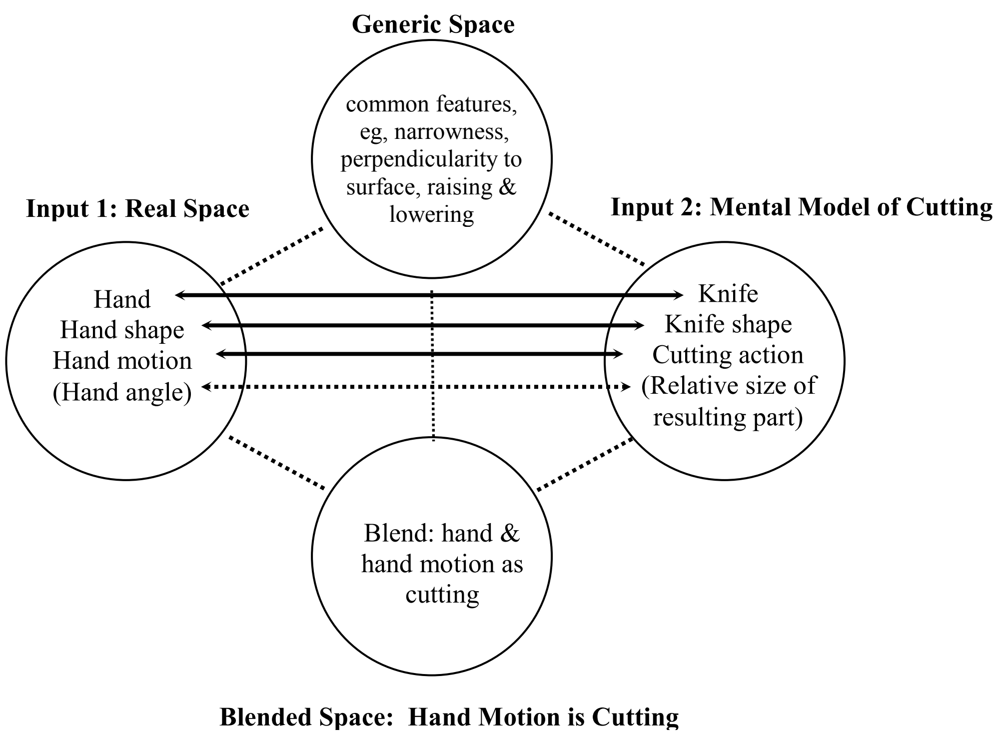

Figure 2. Conceptual blend for the iconic gesture of “cutting”

Example 2: A metaphoric gesture for proof

The next example is drawn from the second study, in which doctoral students talked about mathematical proof, and then collaborated to create one. Figure 4 illustrates a still from a gesture sequence displayed by WG, one of the students in the second study. When asked what kinds of proofs he found difficult or easy, in part of his reply, he said:

"'cause you start figuring out, I'm starting at **point a and ending up at point b**. There's gonna be **some road**//**where does it go through**? And can I show that **I can get through there**?"

WG began the full gesture sequence by closing the fingers of his left hand and touching a location near the top of his thigh ("**point a")**, then opening his right hand and pointing as he moved it away from his body ("**point b**"). He then traced a fairly straight path through the air with his right index finger, returning and pausing briefly after "some road." He then made a small horizontal circle with the same finger, and retraced the path between the origin and end of the gesture.

Figure 4: "Proof is a journey" gesture

The metaphor underlying both the gesture and the speech in this example is clear: WG is conceptualizing proof as a journey. Table 1 summarizes this metaphor (also known as a single-scope conceptual blend).

Source: A Journey	**Target:** A Mathematical Proof

Starting point	Givens
Destination	To prove/conclusion
Possible routes	Possible sequences of statements
"Dead ends"	Sequences that don't result in the conclusion

Table 1: "Proof is a journey" metaphor

The "journey" metaphor was not the only way that this student spoke (and gestured) about proof. Just prior to this example, WG said, "And then the question is, well, can I fill in **those steps** that I have?", while displaying a series of gestures in front of him, with his right hand held horizontal and dropping vertically below itself three times. Although his speech, on its own, might be interpreted as referring directly to a journey ("steps" could refer to walking), his gesture made it clear that the "steps" he was talking about were statements within a proof, written from top to bottom either on a piece of paper, or on a blackboard. The underlying metaphor of a journey is arguably still there, in that the socially common use of "steps" to indicate logical inferences in a proof betrays a grounding in thinking about carrying out a proof in terms of motion or travel. However, the most immediate input space for the conceptual blend is a written inscription, which in turn refers to the recording of a sequence of logical statements.

How do metaphoric gestures work?

Parrill and Sweetser (2004) propose that even metaphoric gestures have an iconic aspect, in that, by means of the hand shapes and motions, they invoke some visual or concrete situation, entity or action. This concrete entity or situation is not arbitrary; rather, it is selected (usually unconsciously) because it provides specific elements and an inferential structure that help support our understanding of the abstraction expressed through the gesture. Thus, a metaphoric gesture involves a sequence of two conceptual mappings, an iconic one between Real Space and the visual/concrete situation (as conceptualized by the speaker), and a second between this conceptual space (the source of the metaphor) and the intended abstract meaning (the target) (Parrill and Sweetser, 2004).

Example 3: A metaphoric gesture for "less than"

The example presented here is drawn from the fractions study, and utilizes conceptual integration to analyze a gesture for an abstract mathematical relationship, rather than a concrete object. In Figure 3, the student, CR, is describing how she had solved a problem comparing two fractions, using the image of sharing a pie. She uses a pointing (or deictic) gesture toward the left to indicate that, in her imagined scenario, some participants would "get less" than others.

Figure 3: "We're each getting **less**"

My hypothesis is that this pointing toward the left is not arbitrary, but is rather based on the conventions used in a specific mathematical representation, the number line. In the number line, numbers become larger in value as you "move" to the right, and smaller as you "move" to the left. In CR's metaphoric physical gesture for "less," the number line both acts as a source domain for the metaphor underlying the gesture, and is itself is a conceptual blend (see Lakoff & Núñez, 2000, for an analysis of the conceptual blend for the number line). Table 2 illustrates the double mapping for CR's metaphoric (and deictic) gesture for "less".

	Iconic Mapping	Metaphoric Mapping
Real Space	*Source*	*Target*
Horizontal space in front of CR	Horizontal number line	A range of quantities
Horizontal space toward the right of center	Numbers toward the right	Increasing quantities
Horizontal space toward the left of center	Numbers toward the left	Decreasing quantities
Moving fingers of right hand toward the left	Pointing from a location on the number line to one located to the left of it	Indicating that one quantity is less than another

Table 2. A double mapping for CR's metaphoric gesture

Here, the "gesture space" in front of the student serves as medium for her to externalize her image of the number line, which forms the source domain allowing her to compare two quantities and indicate that one is less than another. In this way, the gesture (and co-occurring speech) is able to signify an abstract, mathematical relationship, through the use of a conventional physical representation for numbers, the number line.

DISCUSSION

The overall goal of the two studies was to collect and analyze gestures related to particular mathematical topics, fractions in the first case, and proof in the second. The purpose was both to contribute to our understanding of how gestures are used in thinking and communicating about mathematics, and to develop an analytic framework appropriate to understanding gesture (and other modalities) within this specific domain of human cognition.

The framework for analysis of gestures used here draws deeply from conceptual linguistics, in particular, applying the tool of conceptual integration to examine the relationships between gestures, their co-expressive speech, and the meanings of both. Conceptual blends involving the physical environment (including our hands and arms, as well as other tangible objects) can help account for how we generate and interpret gestures that communicate particular ideas. The idea of fraction for many participants evidently involved the notion of cutting or (physically) dividing, as this kind of gesture was very common. Similarly, the notions of part and whole were enacted through the gestures of many students, particularly by evoking images of tangible materials purposely created or selected to "illustrate" fractions. In the more abstract area of proof, gestures drew on metaphors of journeys as well as the physical inscriptions used to create and document proofs. Finally, an example of an apparently simple gesture indicating "less" was linked to a common and conventional representation of numbers, the number line.

The question of how physical materials and inscriptions function to support remembering, using and talking about mathematics, in the context of gesture and other modalities, is one that deserves additional investigation. In the end, gesture may provide an important data source, and conceptual integration a powerful tool, in enhancing our understanding of an embodied mathematics, enacted in the physical world.

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