# THE EXAMINING TWO APPROACHES FOR FACILITATING THE PROCESS OF ARITHMETIC WORD PROBLEMS SOLVING

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## ABSTRACT

This paper focuses on two approaches for facilitating the process of word problems solving. The first approach distinguishes different kinds of occurred errors and the second one recognizes various required and underlying knowledge. The first approach applies Kinfong and Holtan's framework of occurred errors and the second approach applies Mayer’s theory (1992) of underlying knowledge for solving word problems. The main aim of this paper is to examine the relationship between different kinds of occurred errors and various required knowledge in solving Arithmetic word problems. The research methodology is a semi experimental method. The subjects include 89 eight grade students (male and female). The research tools are a descriptive math test regarding six word problems and a directed interview. The results indicate that in solving the arithmetic word problems, increasing students' errors result from lack of linguistic, semantic, structural and communicational knowledge. This study explored that the possible connection between the two approaches for facilitating solving word problems is very important. That is because clarity of this relationship may increase math teachers’ insight about the nature of different kinds of occurred errors and the different aspects of knowledge necessary for solving word problems.

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## INTRODUCTION

The national Council of Teachers of Mathematics (1980) in its ''Agenda for Action '' recommended that problem solving should be the focus of mathematic education. The math word problems among mathematic problems mostly deal with relating the real world situations to mathematical concepts. In fact, such problems help students to use their mathematics knowledge in solving their daily problems. The math word problems are known as instruments which develop the students' ability and talent in solving general problems and also are employed to motivate and increase students' satisfaction with math classes (De Corte, Vershaffel and De Wine, 1989). On the other hand, results obtained from numerous researches indicate that most of the students in various academic grades are facing with many difficulties in their trying to solve such problems. These students are able to use successfully calculation algorithms whereas they are not able to solve word problems which need the same algorithms (Mayer and Hegarty, 1996). The reason for such inability is the fact that solving such problems demands mathematical computations along with other kinds of knowledge including linguistic knowledge, which are required for understanding the problems (Cummins et al, 1988).

For Lave (1992), the word problem is “description of a situation in which one or more question(s) is introduced and solver then obtains its solution using mathematical measures and numerical data incorporated into the problem itself”.

The importance and necessity of learning word problems have been emphasized by many curriculum planers (De corte and et al, 1989). These problems are also considerably used in Iranian text books. But based on our own experiences, as math teachers, and the results of some related studies, it is claim that solving word problems is a hard task for students and they face many difficulties for doing that.

In line with facilitating process of word problems solving, two different research approaches can be recognized. One approach examines students’ occurred errors and the other points to the underlying knowledge for solving these problems.

The first approach which examines student's occurred errors was introduced by Newman (1981), Kinfong and Holtan (1976) and Clements (1980). They believe that when students encounter a problem, they examine different ways in order to find a correct solution. But it misleads them. As it mentioned above, in the field of the arithmetic problem they may add 8 with 5, instead of subtracting 5 from 8. Such answers, which is called student's occurred errors, Possibly the main reason refers to their misunderstanding the problem or may be due to their clerical errors. In any case, it seems that being acquainted with the nature and the reasons of these occurred errors can facilitate the process of solving word problems (Newman 1977, Kinfong and Holtan 1976). In this paper Kinfon and Holtan's framework which is more comprehensive one will be reviewed. Based on Their framework, occurred errors were classified into clerical and computational errors, and, others errors.

1. The clerical errors and Computational errors

The clerical errors maybe happened when the number presented in problem is copied incorrectly in computations, but the operator is chosen correctly. The computational errors include three following types:

a) Computational errors with whole numbers: In solving word problems, students may undertake computations completely or partly incorrect.

b) Computational errors with fractions: In this type of mistakes, students make errors during calculations with fraction similar to computational errors with whole numbers.

c) Computational errors with units: for solving problems, students do not undertake the needed unit transformation or do it incorrectly.

2. Other errors

a) Errors with averages and areas: these kinds of mistakes are related to unfamiliarity with necessary formulation or procedure in solving problems.

b) Use of wrong operation: In this category, error is occurred in using incorrect operator. For example the problem may need the addition operator but another operator has been used in computations.

c) No response error: The error is happened when student present no solution for the problem and leave the problem unsolved. This error takes place in two states:

1) No response, but went on to other problems. 2) No response, did not attempt any further problems.

d) Erred responses offering no clues.

The second approach for solving math word problems, required and underlying knowledge for solving these problems, has been examined and classified by various authors including Schoenfeld (1985) and Mayer (1992). Mayer's theory which seems more reasonable than others is used as the main reference for this approach. Mayer (1992) considers two stages in solving the word problems: Problem comprehension and representation, and, searching for solution and its implementation. He allocated especial knowledge for each stage.

In the present paper, however, along with adoption of Mayer's theory of knowledge, it is tried to present other theories mentioned in the literature in order to make it more applicable, such knowledge are as follows:

1. Linguistic knowledge: This knowledge is used by problem solver to read the problem text. The lack of such knowledge at the beginning of problem solving process stops students' efforts in process of problem solving. Greeno (1985) states that one of the students' weaknesses in solving word problems results from their failure in using linguistic knowledge.

The word problem text includes expressions and numerical quantities with describing special situation for it. Problem solver represents the problem text in his/her mind after reading it.

2. Comprehension knowledge consists all of the knowledge from reading the problem text to the problem comprehension which includes semantic knowledge, structural knowledge and intuition knowledge.

a) Semantic knowledge is a knowledge through which the problem text is comprehended. It means that using this kind of knowledge, data and math expression are not seen as asset of pure words any more. But their meanings are formed through semantic knowledge. Having this knowledge helps students to understand the aim of the problem and to interpret it. Some of students don't interpret the problems correctly. For example if "Mary and john have 5 altogether" it means that "Mary and John each have 5". This misinterpretation led them to construct predictably in coherent problem representations and choose incorrect solution strategies (Cummins 1988).

Understanding the expressions of word problems, with descriptive special situation, required information from real world. This information is a part of the semantic knowledge for problem solver.

b) The intuition knowledge, results from individual, formal and informal, past knowledge, objective experiences, and environment as well as individual capabilities. This knowledge also deals with meaningfulness of problem-related data and information.

After reading the problem, students may examine the correctness or incorrectness of their given answers a long with using their intuition and common sense. Some students who do not have such knowledge only deal with calculations procedures. For instance, consider the following problem:

> "An Army bus holds 30 soldiers. If 1128 soldiers are being bussed to their training site. How many buses are needed?"

Carpenter et al (1983) showed that 70 percent of 13-year old students were able to find their answer as 37 buses and the rest found answer as 37.6 buses. In this case, students used the necessary computational knowledge but they didn't employed their intuition knowledge and common sense to present the meaningful answer.

c) The structural knowledge, relates to schemata, meaning structures and all of mathematical concepts existed in the mind. Schemas are data structures for representing the generic concepts stored in memory (Rumelhart and Norman, 1985). Fischbein (1999) believes that a scheme is also a strategy for solving a certain class of problems. The schemata are knowledge structures which help students to classify problems in order to find the appropriate solution. Therefore schemata and meaning structures of math concepts are taught to student or are created by him/herself. In facing with word problems, the students select a proper method or pattern for their solutions using these schemata and structures. Nesher and Hershkovitz (1994) studied the role of schemata in solving the word problems in his research and found that expert solvers have more ordered and more complete schemata and meaning structures in their hand in solving word problems.

3. Communicational knowledge is a kind of knowledge which links the problem representation to math concepts and structures. The problem solver with such knowledge is able to select the appropriate schema from the math concepts in order to find the relevant solution. In fact, after understanding the problem, problem solver examines some ways through which it is possible to find coordination between the situation described in the problem and appropriate math concepts and structures. Schoenfeld (1985) in his examining of problem solution emphasizes upon this knowledge as a metacognitive knowledge with control aspect. Lester and Garfallo (1982) also used metacognitive strategies concept instead of relational knowledge concept and confirmed the importance of such knowledge in solving the word problems. For them, these strategies include designing a general approach for problem solving, monitoring the solution advancement, general and local reviewing and evaluating of designs as necessary.

4. Calculation knowledge is a math knowledge which relates to calculations in problem solving. In this classification, doing math operations, procedural skills and numerical computations are distinguished from mathematical concepts.

## Purposes of the study

As mentioned in the introductory section, two different approaches about word problems solving were identified via reviewing the related literature. One approach explores and examines students' occurred errors while the other one recognizes the required knowledge for solving such problems. But, until now, not any research has been done in order to determine such effective relationship between these two approaches. So the main aim of the research is:

Clarifying the relationship between kinds of occurred errors and required knowledge for solving word problems.

## METHODOLOGY

Subjects in this research have been selected 89 seven-grade students (30 female & 59 male) from 4 classes of Arak-Iran middle schools randomly. The main reason for selecting eight grade students as the research subjects was that their textbook contained word problems. The research methodology is a semi experimental method. In this study, two types of tools were used, a pencil and paper test (refer to appendix) and a directed interview. The first research tool contains 6 word problems. To pose the pencil and paper test, all the word problems, of seven grade math textbook were gathered and 30 math teachers evaluated their suitability for representing the above mentioned three categories, based on Likert scales. The six selected problems, for the test, were those which gain more score than others.

The above conducted test, identified students' errors. But to explore the kind of the knowledge that student need and were related to their occurred error, researchers interviewed with students. In addition the procedure was filming for later coding data.

The interviewer discussed was about the problems with incorrect answers. And also he asked the students to solve these problems again. In examining the problems which had been incorrectly solved, the interviewer kept trace of students' solution process in order to find the reasons for error and asked some questions to find their weaknesses in related knowledge. With regard to the framework has been put forward for all kinds of knowledge in the introduction, each of these statements focused on especial aspect of needed knowledge.

The way of determining the kind of necessary knowledge, related committed students' errors was as following:

1. Linguistic knowledge in the case that students were unable to read the text.
2. Semantic knowledge in the case that students were not able to explain the purpose of the problem in their own language.
3. Intuition knowledge in the case that the problem or even students' answers were not reasonable with regard to the real word.

4. Structural knowledge in the case that immediately after reading the problem student did not offered any pattern or way for resolving issue.
5. Communication knowledge in the case that students did not have any reasonable argument for his/her suggested solution.
6. Calculation knowledge in the case that students lacked of calculation and algorithm ability problem solving.

Children's responses in interviews were coded by watching the recorded films. Then for correct responses one point and for incorrect zero point was added.

## RESULTS

After the text examination, students' answers were analyzed. In this analysis, the variety of errors were determined and then for the test word problems the errors collected on the basis of Kinfong’s framework, and then they were organized in Tables 1.

The dispersion analysis of errors presented in Kinfong’s framework for the arithmetic word problems as seen in table (1) showed that students most repeated error was "Error in the use of wrong operation" such that they had used incorrect operator for their solutions. The highest rates of computational errors were respectively the computational errors with fractions and the computational errors with whole numbers.

This study dealt with examining the meaningful relationship between each type of the necessary Knowledge and total errors. And also it determined that why each of the errors individually did happen due to which lack of the needed knowledge respectively.

a) The relationship between each type of knowledge and all the happened errors:

Table (1)

Frequency and percentage errors types in arithmetic word problems

| | Errors types | N | $a_1$ | $b_1$ |
|---|---|---|---|---|
| computational errors | clerical errors | 2 | %2.4 | %0.6 |
| | with whole number | 15 | %18.3 | %4.3 |
| | with fraction | 7 | %21 | %4.8 |
| | with units | | | |
| | Average and area errors | 9 | %10.9 | %2.5 |
| | use of wrong operation | 20 | %24.4 | %5.7 |
| No response | but went on to other problems | 6 | %7.3 | %1.62 |
| | did not attempt any further problem | 7 | %8.5 | %1.98 |
| | Erred response offering no clues | 6 | %7.2 | %1.7 |
| | Total | 82 | %100 | %23.2 |

$a_1$: Percentage of Arithmetic word problems errors

$b_1$: Percentage of total errors

Table (2)

The relation between errors types with needed knowledge in arithmetic word problems

| Errors types | | | language knowledge | | comprehension knowledge | | | | | | Communi cation knowledge | | calculation knowledge | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | semantic knowledge | | intuition knowledge | | structure knowledge | | | | | |
| | | | + | - | + | - | + | - | + | - | + | - | + | - |
| **Arithmetic word problems** | **computational errors** | **clerical errors** | 0 | 2 | 2 | 0 | 2 | 0 | 2 | 0 | 2 | 0 | 1 | 1 |
| | | **with whole number** | 15 | 0 | 4 | 11 | 6 | 9 | 10 | 5 | 6 | 9 | 5 | 10 |
| | | **with fraction** | 17 | 0 | 2 | 15 | 9 | 8 | 4 | 13 | 7 | 10 | 9 | 8 |
| | | **with units** | | | | | | | | | | | | |
| | | **average and area errors** | 9 | 0 | 9 | 0 | 9 | 0 | 1 | 8 | 3 | 6 | 9 | 0 |
| | | **use of wrong operation** | 18 | 2 | 0 | 20 | 2 | 18 | 9 | 11 | 4 | 16 | 20 | 0 |
| | **No response** | **but went on to other problems** | 2 | 4 | 0 | 6 | 6 | 0 | 2 | 4 | 0 | 6 | 0 | 6 |
| | | **did not attempt any further problem** | 2 | 5 | 1 | 6 | 3 | 4 | 1 | 6 | 0 | 7 | 0 | 7 |
| | | **Erred response offering no clues** | 1 | 5 | 0 | 6 | 3 | 3 | 1 | 5 | 2 | 4 | 5 | 1 |

_: frequency of existence a kind of knowledge for each occurred error

+: frequency of lack of any kind of knowledge for each occurred error

Table (3)

The relation between each of knowledge types with the total errors

| | **arithmetic world problem** | |
|---|---|---|
| **knowledge types** | chi-square statistic | Liner relation |
| **Language knowledge** | 50.57 | +* |
| **Semantic knowledge** | 49.57 | +* |
| **intuition knowledge** | 39.57 | 0 |
| **structure knowledge** | 16.25 | +* |
| **Communication knowledge** | 13.16 | +* |
| **calculation knowledge** | 44.98 | 0 |

$P<0.05$

0: There is no meaningful correlation.

+*: Meaningful positive correlation between the expected knowledge that has been led to increased number of errors.

-*: Meaningful negative correlation between the expected knowledge that has been led to increased number of errors.

The $X^2$-test was used to examine the meaningful relationship between each of the necessary knowledge and all happened errors ($p<0.05$). From the $X^2$-test scores in table 3, it is obvious that lack of linguistic, semantic, structural and communicational knowledge in the arithmetic word problems has increased student errors.

b) The relationship between each kinds of errors and the necessary knowledge separately: As shown in table 2 , the reasons for making errors in "use of incorrect operation" in arithmetic word problems were respectively enumerated as lack of semantic, intuition, communicational and structural knowledge.

## DISCUSSION

The goal of this article has been to examine the relationship between different kinds of occurred errors and required knowledge for solving word problems. Examining occurred errors and requiring knowledge are two approaches which have been presented regarding literature for facilitating process of word problems solving.

According to the research results, the most common errors in solving math word problems was the application of “wrong operation”. In the examination of student errors rates in solving the arithmetic word problems, concluded that, there was no significant difference between this research and Kinfong and Holtan’s research. These results suggest that acquiring special knowledge decrease the occurred errors and consequently facilitate process of problem solving. Concerning, the determination of the relationship between variety of errors and the required knowledge in solving the word problems, it was revealed that in solving arithmetic word problems, the lack of linguistic, semantic, structural and communicational knowledge has increased the students' errors.

The findings related to the semantic knowledge are partly due to the fact that some students read the problem, but not for comprehension purposes, but in order for extract some key numbers and operation from its text. In fact, most of them did not understand the problem's content and aims (Henjes, 2007). Some of the students had not proper linguistic knowledge and they were unable to read the problem text.

So the lack of linguistic knowledge results in decrease of ability in semantic knowledge. This result somehow coincide the findings of Clements (1980). He concluded that high rate of students' errors in solving seven grade course's word problems were due to their lack of comprehension, translation, processing skills as well as their negligence. Hershkovitz et al (2003) also, in their study found that in solving the word problems, students used available schemata. But when they did not have any appropriate schema they used a stereotyped solution or did not answer at all.

Teachers must firstly make the students involved in the problem and when they were insured of students' comprehension of the problems; they must provide them the relevant knowledge especially with respect to the occurred errors.

This study explored that the possible connection between the two approaches for facilitating solving word problems is very important. That is because clarity of this relationship may increase math teachers' insight about the nature of different kinds of occurred errors and the different aspects of knowledge necessary for solving word problems.

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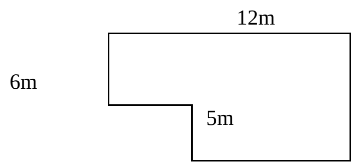
## APPENDIX

1. A publisher published a book in the last year that four-fifth of total its circulation were sold And 1200 volumes were remained in the warehouse, how many books were totally published?

2. A father is 38 years old and his son is 8. How many years should be passed that the father can reach to the age of 3-time older than his son?

3. The roof of a building is like the following picture: we want to insulate the roof with two layers of insulation materials. The width of layers is 1.7 meters. How many meters of layer are needed?




4. The festival committee in your area wishes to prepare a rectangular festival enclosure, with a surface area of 400 $m^2$. The enclosure is to be fenced off with mental fencing costing 3 euros a linear metre. What are the best dimensions for the site, if the cost of the fencing is to be reduced to a minimum? Explain why the dimensions you have chosen are the best.

5. Job offers for pizza delivery workers have appeared in a local newspaper.
   Pizza takeaway A pays each delivery worker 0.6 euros for each pizza delivered and a fixed sum of 60 euros a month. Pizza takeaway B pays 0.9 euros for each pizza delivered and a fixed sum of 24 euros a month.
   Which do you think is the better-paid job?
   Make a decision and explain why your choice is the better?

6. Today the temperature of Arak city is 4°C. its tonight's temperature is 7°C colder. Find the average temperature of Arak city?

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